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A note on shock-shock diffraction

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Whitham's treatment of the 'shock-shock' diffraction of a shock wave advancing into a uniform, quiescent region is extended to a shock wave advancing into a region of uniform flow on the assumption that all velocities are approximately parallel. The result is applied to diffraction of a blast wave by a thin wedge travelling at supersonic speed. The prediction of the pressure on the wedge, just behind the diffracted blast wave, is qualitatively satisfactory.

1. Introduction

Whitham (1957) has treated the diffraction of a shock wave advancing into a uniform, quiescent region by an adaptation of Chisnell's (1957) treatment of shock propagation in a converging channel. We consider here an extension of Whitham's treatment of 'shock-shock' diffraction to the diffraction of a shock wave moving into a region of uniform flow. This uniform flow contributes a velocity component that is tangential to, and conserved across, the diffracted shock, in consequence of which the shock normals (or 'rays') no longer can be regarded as channel walls (as they were in Whitham's original formulation). Chisnell (1965) has recently shown how Whitham's formulation may be modified to accomodate this tangential velocity, and we shall base our formulation on this modification.† (Chester 1960 has considered shock propagation through a converging channel into a region of uniform flow; however, no tangential velocity component arises in this problem.)

We shall apply our result to the diffraction of a blast wave (i.e. a strong shock wave) by a thin wedge travelling at supersonic speed and compare the pressure on the wedge, just behind the blast wave, with that inferred from a complete solution of the boundary-value problem (Smyrl 1963). We shall designate these calculations as approximate and exact, respectively. The corresponding calculations for a stationary wedge have been made by Whitham (1957) and Lighthill (1949).

The results of our comparison are qualitatively satisfactory, but the quantitative error appreciably exceeds that in the corresponding comparison for the stationary wedge. This suggests that Chisnell's generalization of Whitham's rule ((2.3) below) may be less satisfactory for shock propagation into a region of

† I am indebted to Dr Chisnell for bringing this modification to my attention after reading an earlier version of the present paper that was based on Whitham's original rule [$M = 0$ in (2.3) below]. Chisnell's modification effected a substantial improvement in the comparison of figure 6 below.

uniform flow than for propagation into a quiescent region, although further investigation would be required to support any definitive conclusion.

Both the approximate and exact treatments of the travelling wedge depend on whether the incident blast wave intersects the original bow wave of the wedge outside or inside of the sonic circle associated with the initial penetration of the blast wave (cf. figures 1 and 6 in Smyrl's paper). We shall consider only the former possibility, in which case the first step in both treatments is to calculate the diffraction of the incident blast wave by a Mach wave (i.e. a weak shock wave). The results of this preliminary calculation are implicit in Smyrl's analysis, at least for $\gamma = 7/5$, but we shall consider it briefly (in § 3 below) and obtain explicit results for the angle through which the blast wave is turned and for the Mach number of the diffracted blast wave.

2. Extension of Whitham's rule

Assuming that all velocities are approximately parallel (so that we may neglect the squares of the angles of inclination), we consider an approximately uniform, plane shock wave that moves with relative velocity $m a_2$ into a uniform,

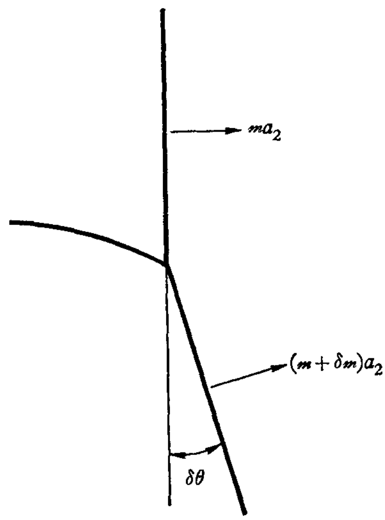

FIGURE 1. Shock-shock interaction, with arrows indicating velocities of original and diffracted shocks relative to medium on right.

plane flow of velocity Ma_2 and sonic speed a_2. We require the change in m, say δm, associated with a small change $\delta\theta$ in the angle of inclination of the shock ($\delta\theta > 0$ implies that the shock is locally concave with respect to the uniform flow on the right, as shown in figure 1).

We can deduce from purely kinematical considerations [cf. Whitham 1957, equation (18)] that

$$\delta\theta = (-\delta A/A)^{\frac{1}{2}}[\delta m/(M+m)]^{\frac{1}{2}}, \tag{2.1}$$

where δA denotes the incremental change in A, the distance between a pair of rays orthogonal to the shock ($\delta A = A_1 - A_0$ and $\delta m = M_1 - M_0$ in figure 6 of Whitham's paper).

Invoking our hypothesis of approximately parallel flow and assuming a perfect gas, we can calculate the local particle velocity, pressure, sonic speed, and density behind the shock from the Rankine–Hugoniot equations

$$q = a_2[M+2(\gamma+1)^{-1}(m-m^{-1})], \tag{2.2a}$$

$$p = p_2[1+2\gamma(\gamma+1)^{-1}(m^2-1)], \tag{2.2b}$$

$$a^2 = a_2^2[2\gamma m^2-(\gamma-1)][(\gamma-1)m^2+2]/(\gamma+1)^2 m^2, \tag{2.2c}$$

$$\rho = \gamma p/a^2, \tag{2.2d}$$

where γ denotes the specific-heat ratio. Following Chisnell (1964), we now suppose that the characteristic relation

$$\delta p+\rho a\delta q+\frac{\rho a^2 q}{(q+a)}\left[1-\left(\frac{Ma_2}{q}\right)\right]\frac{\delta A}{A} = 0 \tag{2.3}$$

holds just behind the shock. Substituting (2·2) into (2·3), we obtain

$$-\frac{\delta A}{A} = 2\left[\frac{m+L(m)M}{(m^2-1)K(m)}\right]\delta m, \tag{2.4}$$

where
$$K(m) = 2[1+2(\gamma+1)^{-1}(\mu^{-1}-\mu)]^{-1}(1+m^{-2}+2\mu)^{-1}, \tag{2.5}$$

$$\mu(m) = [(\gamma-1)m^2+2]^{\frac{1}{2}}[2\gamma m^2-(\gamma-1)]^{-\frac{1}{2}}, \tag{2.6}$$

and
$$L(m) = (\gamma+1)\{2(1-m^{-2})+[2\gamma-(\gamma-1)m^{-2}]^{\frac{1}{2}}[(\gamma-1)+2m^{-2}]^{\frac{1}{2}}\}^{-1}. \tag{2.7}$$

Setting $M = 0$ in (2.4) yields Chester's original result, as utilized by Whitham. Chester's function $K(m)$ decreases monotonically from $\frac{1}{2}$ at $m = 1$ to 0·3941 at $m = \infty$ (for $\gamma = 7/5$). The function $L(m)$ decreases from 1 at $m = 1$ through a very flat minimum of approximately 0·779 near $m = 3$ to 0·7848 at $m = \infty$ (for $\gamma = 7/5$); it is within 2 % of 0·80 for $m \geqslant 1{\cdot}5$ or 0·7 % of 0·78 for $m \geqslant 2$.

Substituting (2.4) into (2.1), we obtain

$$\frac{\delta m}{\delta\theta} = G(m, M) = \left\{\frac{(m+M)(m^2-1)K(m)}{2[m+L(m)M]}\right\}^{\frac{1}{2}}, \tag{2.8}$$

which is the required extension of Whitham's 'shock-shock' relation as $\delta\theta \to 0$. We observe that (2.8) reduces to Whitham's [1957, equation (22)] result if $M/m \to 0$.

We could extend the foregoing results in various ways. Thus, we can generalize Whitham's (1957) differential equation for shock wave propagation to

$$\frac{\partial}{\partial\alpha}\left[(M+m)^{-1}\frac{\partial A}{\partial\alpha}\right]+\frac{\partial}{\partial\beta}\left[A^{-1}\frac{\partial m}{\partial\beta}\right] = 0 \tag{2.9}$$

and integrate (2.4) to obtain

$$A = k(\beta)f(m, M), \tag{2.10a}$$

$$f(m, M) = \exp\left\{-2\int^{m}\left[\frac{m+L(m)M}{m^2-1}\right]\frac{dm}{K(m)}\right\}. \tag{2.10b}$$

However, the application of these more general results would be severely limited by our assumption that all velocities are approximately parallel.

3. Blast-wave, Mach-wave interaction

Referring to figure 2, we now consider the two-dimensional diffraction of a blast wave (strong shock) by a Mach wave (weak shock) that emanates from the apex of a thin wedge of semi-apex angle α in an otherwise undisturbed flow of supersonic speed U. Choosing a reference frame in which the wedge is at rest, we suppose the blast wave to advance into the region of undisturbed flow, say

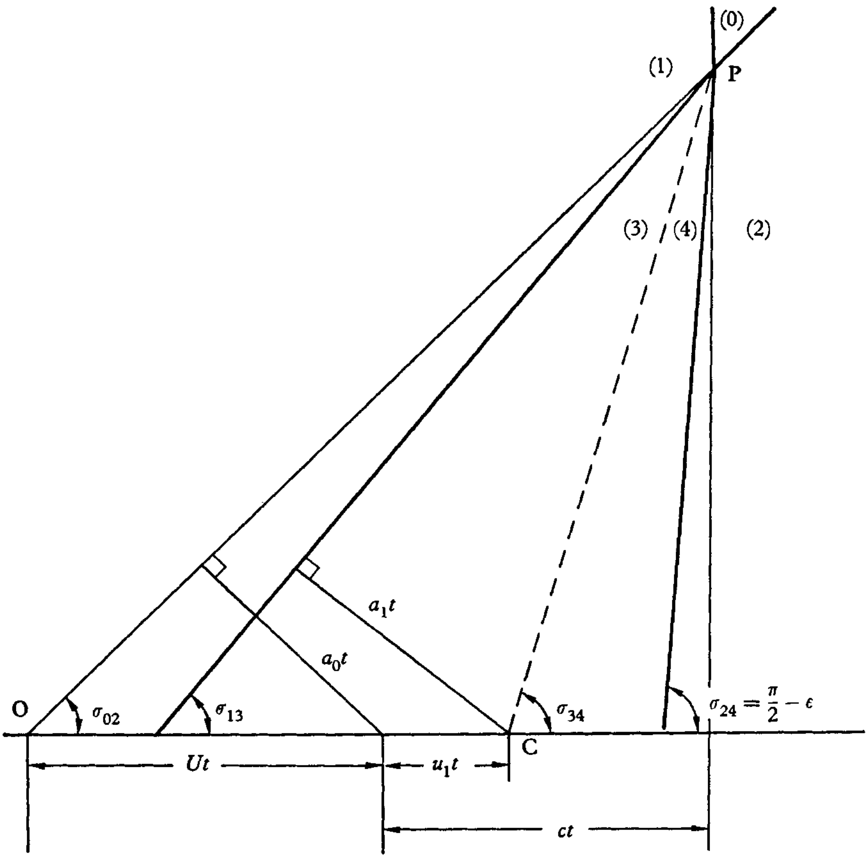

FIGURE 2. Diffraction of blast wave (01) by Mach wave (02).

region 0, with relative speed c and to be normally incident on the apex of the wedge at $t = 0$. We define those regions in which the fluid has been disturbed only by the blast wave or only by the Mach wave as regions 1 and 2, respectively. Given U, α, c, and the state of the fluid in region 0, we seek the angle $-\epsilon$ through which the incident blast wave is turned and the speed c' with which the diffracted blast wave advances into region 2.

Let 02, extended through P to O, be the original, undisturbed Mach wave, where P is the shock intersection at time t and O the intersection at $t = 0$, 02 be the undisturbed Mach wave at time t, 01 be the incident blast wave, 13 be the diffracted Mach wave, and 24 be the diffracted blast wave. The changes across 02 and 13 are $O(\alpha)$ and, by hypothesis, small, whereas the changes across 01 and

24 need not be small. Regions 3 and 4 must be separated by the contact discontinuity 34. We approximate this contact discontinuity by CP in figure 2 by virtue of the fact that its velocity differs from the particle velocity in region 1, say $U+u_1$, only by a term of $O(\alpha)$.

We can determine the foregoing pattern completely by invoking the weak-shock equations across 02 and 13, the Rankine–Hugoniot equations across 01 and 24, and the requirements of continuity of normal velocity and pressure

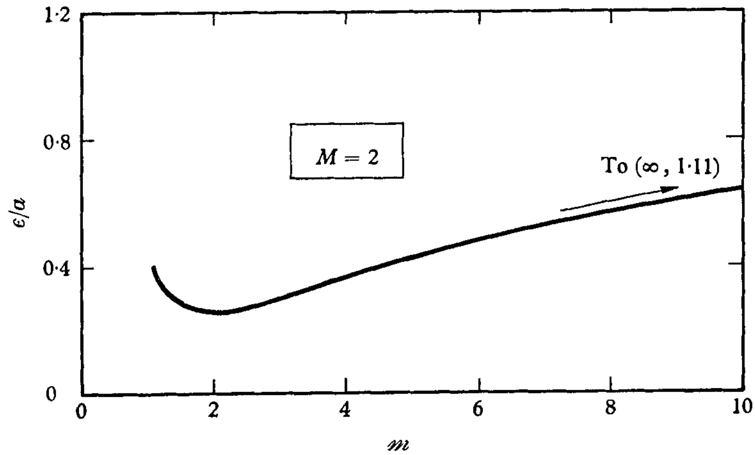

FIGURE 3. The ratio of the shock diffraction angle ϵ to the Mach-wave deflexion angle α for $M = 2$ and $\gamma = 7/5$.

across 34. We shall omit the details of the calculation, since the results are not new (cf. Smyrl 1963), although they do not appear to have been given explicitly. Letting

$$m = c/a_0 \quad \text{and} \quad M = U/a_0, \tag{3.1a,b}$$

we find that

$$\begin{aligned}\epsilon/\alpha = {} & M\{m(M+m)+\tfrac{1}{2}m^{-3}(M+\kappa m)\,[1+\lambda(M^2-1)+(M/m)]^{-1} \\ & \times[(1-\kappa)^{-1}(M+m)^2(m^2+1)-m^2(m^2-1)(M^2-1)]\}^{-1} \\ & \times\{M+m-\tfrac{1}{4}(\gamma+1)M(1-\kappa m^2)+m^{-2}(M+\kappa m)[1+\lambda(M^2-1)+(M/m)]^{-1} \\ & \times[\tfrac{1}{4}(\gamma+1)\,m(2m+M-M^2m)-\tfrac{1}{2}(\gamma-1)(M+m)^2(1-\kappa)^{-1}]\},\end{aligned} \tag{3.2}$$

where

$$\kappa = 2(\gamma+1)^{-1}(1-m^{-2}) \tag{3.3}$$

and

$$\begin{aligned}\lambda = {} & (1-\kappa)^{\frac{3}{2}}(m^2-1)\{(1+\gamma\kappa m^2)^{\frac{1}{2}}[(M+m)^2 \\ & -(1-\kappa)(M^2-1)(m^2-1)]^{\frac{1}{2}}-(1-\kappa)^{\frac{1}{2}}m(M+m)\}^{-1}.\end{aligned} \tag{3.4}$$

We observe that λ may be complex, in which case (figure 6 of Smyrl's paper) the shock wave intersects the Mach wave within the sonic circle of figure 2; however, we shall not consider this case. We also obtain

$$\begin{aligned}m' = c'/a_2 = {} & m+(M^2-1)^{-\frac{1}{2}} \\ & \times\{M[1-\tfrac{1}{2}(\gamma-1)Mm]\,\alpha-(M+m)\,\epsilon\}+O(\alpha^2),\end{aligned} \tag{3.5}$$

for the Mach number of the diffracted shock wave 24 relative to the fluid in region 2.

Numerical results for ϵ/α, with $\gamma = 7/5$, are plotted in figures 3 and 4. A more detailed derivation of the result, together with a rather unsuccessful attempt to obtain it with an *ad hoc* adaptation of Whitham's approximation, has been given elsewhere (Miles 1963).

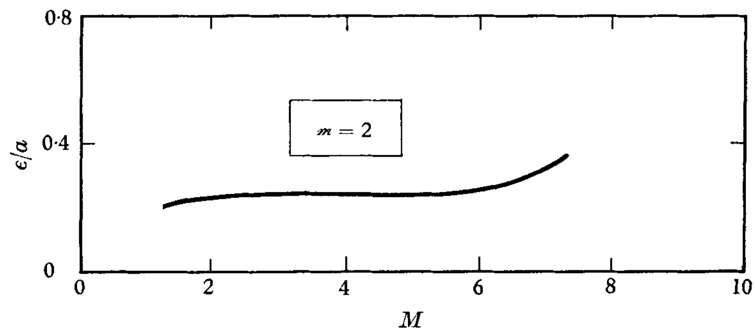

FIGURE 4. The ratio of the shock diffraction angle ϵ to the Mach-wave deflexion angle α for $m = 2$ and $\gamma = 7/5$.

4. Shock-wave, sonic-circle interaction

The diffracted shock wave 02 actually remains straight only above the sonic circle of radius $a_1 t$ and centre C in figure 2. Below its intersection with this circle, it is diffracted as a curved shock that must meet the wedge normally. We shall approximate this curved shock by the plane shock AB of figure 5. The pressure just behind it then is uniform and equal to that at the point A on the wedge. Invoking (2.2*b*), we obtain

$$p_A = p_2[1 + 2\gamma(\gamma+1)^{-1}(m''^2 - 1)], \tag{4.1}$$

where m'' denotes the Mach number of AB.

Invoking the weak-shock equations (i.e. linearized airfoil theory), we obtain

$$p_2 = p_0[1 + (\gamma M^2/B)\,\alpha], \quad B = (M^2-1)^{\frac{1}{2}}. \tag{4.2a, b}$$

Substituting (4.2) into (4.1), remarking that $m'' - m = O(\alpha)$, and introducing

$$p_1 = p_0[1 + 2\gamma(\gamma+1)^{-1}(m^2-1)] \tag{4.3}$$

for comparison, we can transform (4.1) to

$$p_A = p_1\{1 + \gamma[(M^2/B) + S(m, M)]\,\alpha + O(\alpha^2)\}, \tag{4.4}$$

where

$$S(m, M) = 4m(2\gamma m^2 - \gamma + 1)^{-1}(m'' - m)/\alpha. \tag{4.5}$$

We now calculate m'' from (2.8) according to

$$m'' = m' + G(m', M)(\epsilon + \alpha) + O(\alpha^2) \tag{4.6a}$$

$$= m' + G(m, M)(\epsilon + \alpha) + O(\alpha^2), \tag{4.6b}$$

where m' is given by (3.5). Substituting the resulting expression for $m'' - m$ into (4.5), we obtain

$$S(m, M) = 4m(2\gamma m^2 - \gamma + 1)^{-1}\{[1 + (\epsilon/\alpha)]\,G(m, M) \cdot \\ + MB^{-1}[1 - \tfrac{1}{2}(\gamma-1)\,Mm] - (M+m)\,B^{-1}(\epsilon/\alpha)\}. \tag{4.7}$$

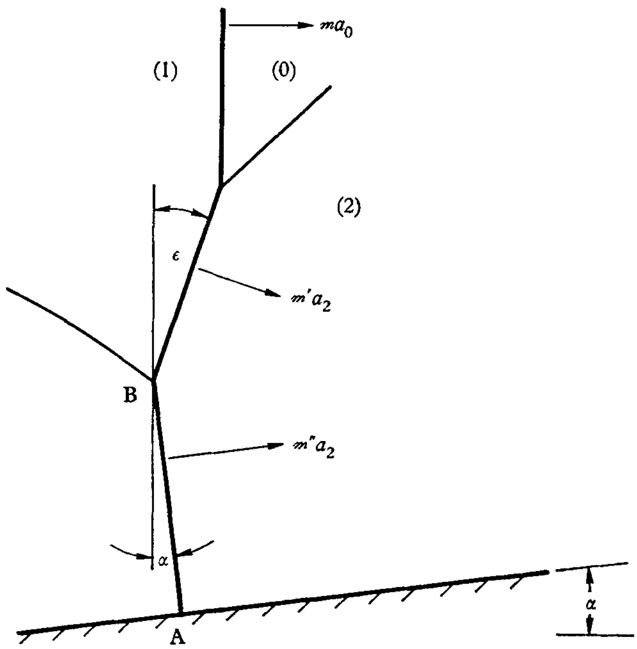

FIGURE 5. The assumed pattern for diffraction of a blast wave by a moving wedge. That portion of the pattern above B, comprising the incident blast wave 01, the Mach wave 02, and the upper portion of the diffracted blast wave, is as in figure 2.

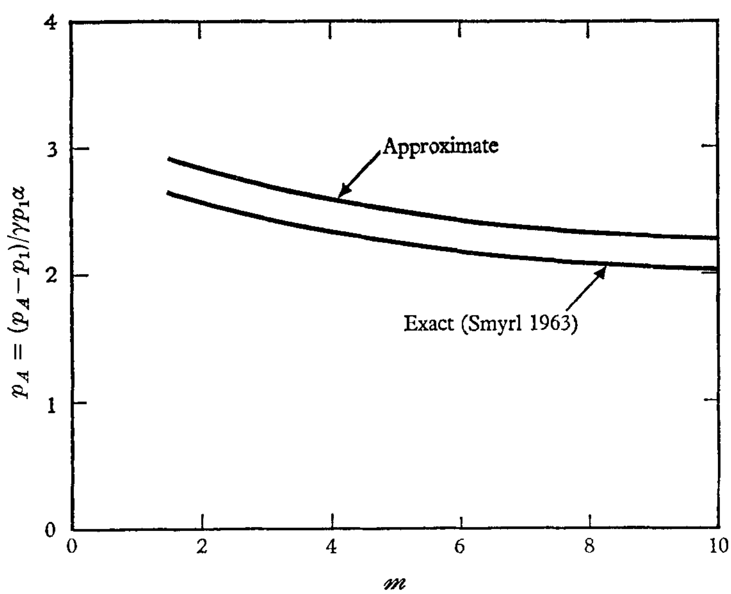

FIGURE 6. The relative pressure on the wedge, just behind the diffracted blast wave, for $M = 2$ and $\gamma = 7/5$.

We can compare the relative effects of the Mach wave and the sonic circle on the diffraction of the blast wave through the relative increments of either angle of inclination or Mach number. The blast wave is turned through angles of magnitude ϵ and $\epsilon+\alpha$ by the Mach wave and the sonic circle, respectively, and the ratio $\epsilon/(\epsilon+\alpha)$ varies between roughly 0·2 ($m=2$) and 0·5 ($m=\infty$) for $M = 2$. The corresponding ratio of incremental Mach numbers is

$$\frac{m'-m}{m''-m'} = \frac{MB^{-1}[1-\frac{1}{2}(\gamma-1)\,Mm]-(M+m)\,B^{-1}\,(\epsilon/\alpha)}{[1+(\epsilon/\alpha)]\,G(m,M)}, \tag{4.8}$$

which varies between roughly $-0·2$ ($m=1·5$) and $-1·2$ ($m=\infty$) for $M = 2$ and $\gamma = 7/5$.

The quantity $S+(M^2/B)$, designated as p' by Smyrl, is compared with Smyrl's result for $M = 2$ and $\gamma = 7/5$ in figure 6. We conclude from this comparison that our approximation is qualitatively satisfactory.

This work was done while the author was a consultant to the Aerodynamics and Propulsion Research Laboratory, Aerospace Corporation, El Segundo, California.

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